# Prediction of Polydispersed Fluidized Bed System Properties: A New Approach

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*A new approach for estimating the system properties of polydispersed liquid-solid fluidized bed systems, based on the concept of departure from monodispersed systems, is discussed. The proposed generalized correlation based on experimental data on particulate fluidization can predict minimum fluidization velocity, minimum elutriation velocity, onset of semifluidization velocity, and free-settling velocity with an accuracy of ±25.7%. The correlation is valid for heterogeneous systems comprising solids of various sizes and/or densities.*

## Introduction

To study any fluidized bed process, various system properties will usually be estimated, for example, minimum fluidization velocity, $u_{mf}$; minimum elutriation velocity, $u_{me}$; onset of semifluidization velocity, $u_{osf}$; maximum semifluidization velocity, $u_{msf}$ or the free settling velocity, $u_t$. Such characteristic properties are dependent on the physical characteristics of the bed of solid particles (that is, particle diameter, solid density) as well as those of the fluid (that is, density, viscosity). These system properties are thus governed by the physical properties of the solid and fluid and are not affected by a change in the superficial fluid velocity. The latter can vary from a very low value representing the fixed bed condition to a very high value, which represents the condition of elutriation or entrainment. Such properties, nevertheless, give an idea of the fluid flow requirements which is instrumental in finalizing the capacity of a pump or blower.

Attempts have been made in the past to correlate system properties with some characteristics dimensionless groups, representing average diameter and average density of the solid particles comprised in the bed of the fluidizing column (Ganguly, 1982, 1990). For all practical purposes, such beds are heterogeneous in nature and consist of solid particles of various sizes and/or densities (polydispersed/multicomponent).

The number of such empirical correlations is fairly large. The nature of such correlations in some cases is complicated, often leading to tedious trial-and-error solutions (Barnea-Mednick, 1975; Limas-Ballesteros et al., 1982; Chiba et al., 1979; Clift et al., 1975; Zigrang-Sylvester, 1981). These correlations also differ with regard to their accuracies and ranges of applicability. The task of choosing the best possible correlation to estimate a particular fluidized-bed system property is therefore apt to pose a problem.

A different approach has been applied recently (Ganguly, 1989) using means from which the average system properties can be evaluated from a knowledge of similar properties of the individual components that constitute the heterogeneous bed of solids.

Instead of selecting different equations for predicting different system properties, an attempt has ben made in this article to predict such properties by a single unified correlation.

The concept stems from the well-known theory of corresponding states in thermodynamics (Glasstone, 1969) which states that substances, equally displaced from the standard state (generally critical state), behave identically. Such a shift is represented by the term "reduced condition" which is the ratio of the existing to the critical property, for example, $T_r = T/T_c$, $P_r = P/P_c$. The present postulate tries to quantify the change in the static properties of a bed caused in some degree to heterogeneity, by some reduced condition. That condition is defined by the ratio of average property of heterogeneous bed to the property of homogeneous bed. It is also postulated that a bed of solid particles whose physical characteristics are shifted to some extent from those of monodispersed (homogeneous) system will also produce the same change in the reduced variables, for example, $u_{t_R}$, $u_{mf_R}$, $u_{osf_R}$. Such departure in the physical properties of the solids can be expressed by reduced Archimedes number and reduced bulk density denoted by $Ar_R$ and $\rho_{B_R}$ respectively. The reduced system property, $P_R$, is expected to be related with the groups $Ar_R$ and $\rho_{B_R}$. The nature of this correlation is established in this article.

## Theoretical Properties

The behavioral response of a polydispersed system has been found to be distinctly different from a monodispersed system with respect to various dynamic properties, for example, $u_{mf}$ (Kumar-SenGupta, 1974) or $u_t$ (Coulson-Richardson, 1962). The magnitude of any such property, $u_{mf}$, for example, for a bed comprising coarse and fine particles will be higher than that for a bed, consisting of fine particles only (Ganguly, 1976; Vaid, 1978).

Considering the fact that the system properties of monodispersed systems, $P_i^*$, vary according to the variation of the physical properties of the same system, expressed by the Archimedes number, $Ar_i^*$, and the bulk density, $\rho_{B_i}^*$ (considering the surface irregularities or shapes of the solid particles), the concept of reduced properties has been brought in. As the reduced condition of a substance in thermodynamics reveals the degree of departure from the critical state (Glasstone, 1969), the present concept also enables quantitative evaluation of the degree of departure of the system properties from those of the monodispersed systems which are considered to be the reference states in the present work. Such departure in a similar manner is expressed by the ratio of the average property (dynamic or static) of the polydispersed system to the static or dynamic property of a monodispersed system, which is termed the reduced property of the bed. Accordingly, reduced Archimedes number, $Ar_R$, is defined as the ratio of the Archimedes number for the polydispersed bed, $\overline{Ar}$, to that for the monodispersed bed with respect to $i$th species of solid particles, $Ar_i^*$. For any heterogeneous bed, the Archimedes number, $\overline{Ar}$, is evaluated at the harmonic mean diameter $\overline{d}_p$, and the weighted average density of the solids, $\overline{\rho}_s$, defined as:

$$\overline{d}_p = 1/\Sigma(x_i/d_{pi}^*)$$
$$\overline{\rho}_s = \Sigma(x_i\rho_{si}^*) \quad (1)$$

Irrespective of the type of the system property, that is, $u_t$, $u_{me}$, $u_{mf}$, $u_{osf}$ or $u_{msf}$, the reduced system properties, $P_{R_i}$ ($=\overline{P}/P_i^*$), with respect to any of the species $i$, have been found to be correlated with the reduced physical properties of the heterogeneous bed, given by $(Ar_R)_i$ and $(\rho_{B_R})_i$, by means of any of the following equations:

$$P_{R_i} = A\,(Ar_R)_i^{a_1}(\rho_{B_R})_i^{a_2} \quad (2)$$

$$P_{R_i} = 1 + B[\{(Ar_R)_i/(\rho_{B_R})_i\}^{b_1} - 1] \quad (3)$$

$$P_{R_i} = 1 + C[(Ar_R)_i^{c_1}(\rho_{B_R})_i^{c_2} - 1] \quad (4)$$

$$P_{R_i} = D + D_1(Ar_R)_i + D_2(\rho_{B_R})_i + D_3(Ar_R)_i(\rho_{B_R})_i + D_4(Ar_R)_i^2 + D_5(\rho_{B_R})_i^2 \quad (5)$$

From the definition of the reduced properties of the fluidized bed it is clear that when $(Ar_R)_i = 1$ and $(\rho_{B_R})_i = 1$, the bed is homogeneous in nature. Consequently, the value of $(P_R)_i$ becomes equal to 1.0. This situation defines the essential condition that must be fulfilled by any correlation, based on the postulate of reduced property of fluidized bed.

## Experimental Studies

The experimental setup used for the present study has been described elsewhere (Ganguly, 1980). Essentially, it consisted of a cylindrical fluidizing column, made of plexiglass, which had an inside diameter of 49.2 mm and a height of 930 mm. The lower end of the column fused with a conical, metallic calming section which was filled with glass balls of different diameters to ensure proper distribution of liquid inside the column. The entrained solids were filtered by a strainer, and the liquid stream was recycled to the reservoir. A scale was fitted along the side of the fluidizing column to record the expanded bed height directly (used to determine the settling velocity).

The author's data (Ganguly, 1976) for $u_t$ (or $Re_t$) and $u_{me}$ (or $Re_{me}$) have been processed along with the data of Vaid (Vaid, 1978) for $u_{be}$, $u_{me}$, $u_{mf}$, $u_{osf}$ and $u_{msf}$ to test the validity of the present postulate. All the experiments have, however, been confined to liquid-solid (particulate) fluidization, using water as the working fluid with a density of 1.00 g/cm$^3$ and a viscosity of 0.95 cp. All the system properties except $u_t$ have been evaluated from the conventional plots of $\Delta P_a$ vs. $u_f$, as shown schematically in Figure 1.

Various solids used in this study include coal, graphite, sand, limestone, chalcopyrite and magnetite. All the solids with the exception of sand have been crushed primarily in a double roll crusher, followed by grinding in a ball mill. The ground matrials were sieved into various close-cut size fractions. The physical properties solid density and bulk density as well as the diameter of the different size fractions are shown in Table 1. Since bulk density takes into account the shape (or irregularity) of solid particles, the shape factor or sphericity of the solids has not been considered.

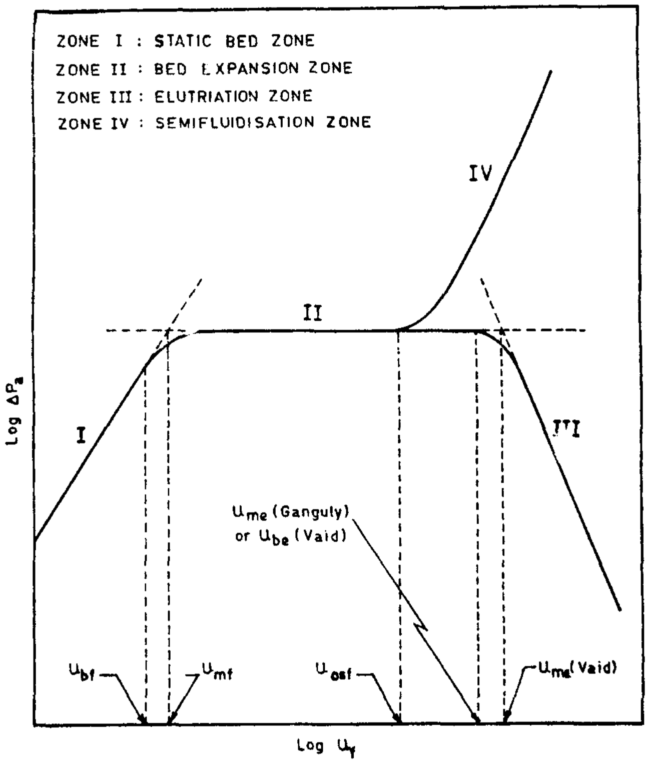


**Figure 1. Variation of actual bed pressure drop with superficial fluid velocity.**

Table 1. Physical Properties of Various Solid Investigated

| Solid | Mesh BSS | $d^*_{p_i}$, cm | $\rho^*_{B_i}$, g/cm$^3$ | $\rho^*_{s_i}$, g/cm$^3$ |
|---|---|---|---|---|
| Coal | 30/40 | 0.0472 | 0.726 | 1.67 |
| | 40/52 | 0.0367 | 0.741 | |
| | 52/60 | 0.0274 | 0.757 | |
| | 60/72 | 0.0231 | 0.786 | |
| | 72/80 | 0.0194 | 0.816 | |
| Graphite | 30/40 | 0.0472 | 1.142 | 2.36 |
| | 40/52 | 0.0367 | 1.171 | |
| | 52/60 | 0.0274 | 1.189 | |
| | 60/72 | 0.0231 | 1.254 | |
| | 72/80 | 0.0194 | 1.257 | |
| Sand | 8/10 | 0.1829 | 1.2728 | 2.66 |
| | 10/14 | 0.1439 | 1.2802 | |
| | 16/20 | 0.0947 | 1.3168 | |
| | 20/30 | 0.0697 | 1.3355 | |
| | 30/40 | 0.0472 | 1.360 | |
| | 40/52 | 0.0367 | 1.382 | |
| | 52/60 | 0.0274 | 1.395 | |
| | 60/72 | 0.0231 | 1.402 | |
| | 72/80 | 0.0194 | 1.447 | |
| Limestone | 30/40 | 0.0472 | 1.476 | 2.72 |
| | 40/52 | 0.0367 | 1.480 | |
| | 52/60 | 0.0274 | 1.485 | |
| | 60/72 | 0.0231 | 1.508 | |
| | 72/80 | 0.0194 | 1.518 | |
| Chalcopyrite | 30/40 | 0.0472 | 1.311 | 3.00 |
| | 40/52 | 0.0367 | 1.325 | |
| | 52/60 | 0.0274 | 1.334 | |
| | 60/72 | 0.0231 | 1.358 | |
| | 72/80 | 0.0194 | 1.387 | |
| Magnetite | 30/40 | 0.0472 | 1.497 | 4.33 |
| | 40/52 | 0.0367 | 1.544 | |
| | 52/60 | 0.0274 | 1.601 | |
| | 60/72 | 0.0231 | 1.660 | |
| | 72/80 | 0.0194 | 1.685 | |

## Definitions

### *Settling velocity, $u_t$*

The free settling velocities of solids for mono-, as well as polydispersed systems have been estimated in the conventional manner, for example, from the plots of voidage, $\epsilon$, vs. superficial fluid velocity, $u_f$, reading the values of $u_f$ at $\epsilon = 1.0$. The values of bed voidage have been calculated from the data on expanded bed height, $L_e$, recorded directly from the scale as:

$$\epsilon = 1 - \frac{L_s}{L_e}$$

$$= \left(\frac{W_f}{\pi/4\, D_t^2 \bar{\rho}_s L_e}\right)$$

$$= 1 - (1.273\, W_f / D_t^2 \bar{\rho}_s L_e) \tag{6}$$

### *Minimum elutriation velocity, $u_{me}$*

Minimum elutriation velocity, $u_{me}$, has been defined (Ganguly, 1982) as the fluid velocity at which the solids begin to leave the system. This is determined graphically from the plot of $\Delta P_a$ vs. $u_f$ (Figure 1) as the break point at which $\Delta P_a$ begins falling due to the gradual depletion of solids in the column. Such velocity has been alternatively termed (Vaid, 1978) as the beginning of elutriation velocity, $u_{be}$, while the minimum elutriation velocity, $u_{me}$, was defined (Vaid, 1978) as the point of intersection between the lines, representing the zones of fluidization (constant $\Delta P_a$) and elutriation (falling $\Delta P_a$). It has also been shown (Ganguly, 1982) that the static bed height, $h_s$, which is the initial height occupied by the solid particles inside the fluidizing column (that is, the height of the solid bed when $u_f = 0$), affects the value of $u_{me}$. All the runs have, therefore, been confined to a constant aspect ratio (the ratio between the static bed height and the inside diameter of the column) of 2.0.

### *Onset or minimum semifluidization velocity, $u_{osf}$*

The phenomenon of semifluidization occurs when a constraint is placed axially inside the fluidizing column at any height above the static bed. The minimum or onset semifluidization velocity is defined as the superficial fluid velocity at which the solid particles begin to form a bed below the constraint. This situation gets reflected in the $\Delta P_a$ vs. the $u_f$ plot at the point at which the pressure drop shoots up with an increase in fluid velocity (Figure 1). Studies of Vaid (Vaid, 1978) have been considered to arrive at the present correlation.

### *Maximum semifluidization velocity, $u_{msf}$*

In the process of semifluidization, the packed bed height below the constraint goes on increasing as the fluid velocity gradually increases beyond $u_{osf}$. A situation is finally achieved when the entire solids, comprising the static bed, get lifted to the bottom of the constraint (height of the packed bed becomes equal to that of the static bed). The fluid velocity under such a condition is termed as the maximum semifluidization velocity, which is determined graphically from the plot of $h_p/h_s$ vs. fluid velocity by extrapolating $h_p/h_s$ to 1.0 (not shown in Figure 1). For an obvious reason, the value of this group is zero at condition $u_f < u_{osf}$ (since $h_p = 0$).

### *Minimum fluidization velocity, $u_{mf}$*

The minimum fluidization velocity, $u_{mf}$, is determined experimentally from the plot of actual pressure drop across the bed of solids, $\Delta P_a$, vs. superficial fluid velocity, $u_f$. Conventionally, the point of intersection between the straight lines, representing the static bed zone and the fluidization zone, gives the value of $u_{mf}$. Although such break points have been shown (Vaid, 1978) to be sharp in particulate fluidization for monodispersed systems, the sharpness is lost (Couderc, 1985; Vaid, 1978) for polydispersed (multicomponent or heterogeneous) systems and the changeover from static to fluidized bed zone is gradual in nature (Figure 1). Studies on $u_{mf}$ have been examined from Vaid (1978) for various heterogeneous systems.

## Results

Reaching the generalized correlation for the reduced system properties, the conventional multiple linear regression or nonlinear regression has been adopted for 260 of the author's data points on $u_t$ and $u_{me}$, and for 551 data points (Vaid, 1978) on $u_{mf}$, $u_{be}$, $u_{me}$, $u_{osf}$ and $u_{msf}$. Based on this, the values of the coefficients and exponents of Eqs. 2 to 5 were evaluated to provide the following results:

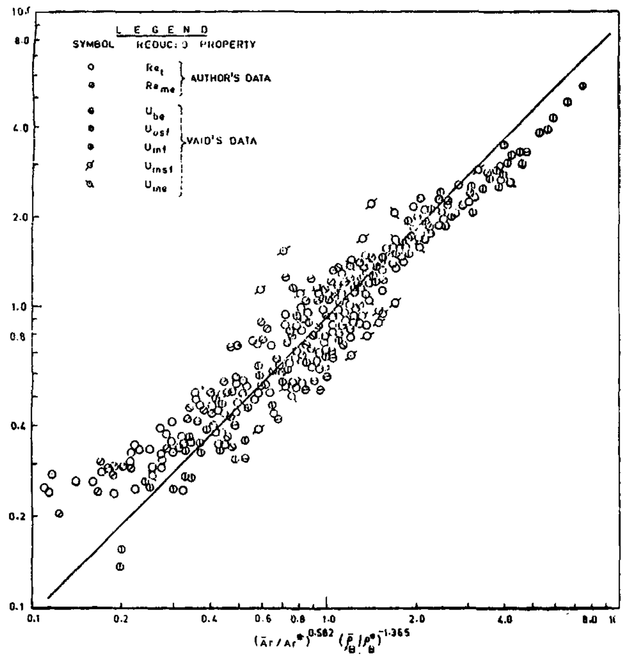


**Figure 2. Correlation plot for Eq. 7.**

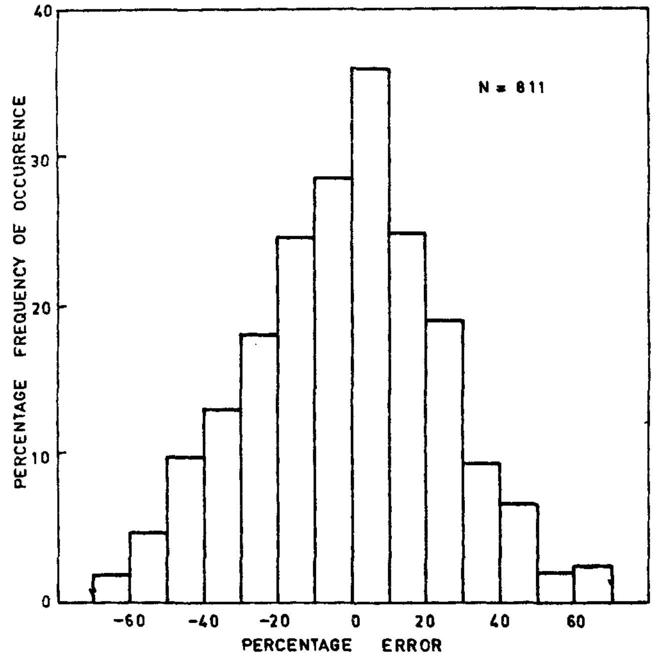


**Figure 3. Frequency distribution of percentage errors.**

$$A=0.9393,\ a_1=0.582,\ a_2=-1.365$$
$$B=0.7264,\ b_1=0.702$$
$$C=0.7277,\ c_1=0.697,\ c_2=-0.894$$
$$D=-1.4662,\ D_1=4.698,\ D_2=1.883$$
$$D_3=1.000,\ D_4=0.0103,\ D_5=-4.210$$

Putting these values in Eqs. 2 to 5 and with $x_1=(Ar_R)_i$ and $x_2=(\rho_{B_R})_i$, the equations reach the final form:

$$P_{R_i}=0.939(x_1)^{0.582}(x_2)^{-1.365} \tag{7}$$

$$P_{R_i}=1+0.726[(x_1/x_2)^{0.702}-1] \tag{8}$$

$$P_{R_i}=1+0.728[(x_1)^{0.697}(x_2)^{-0.894}-1] \tag{9}$$

$$P_{R_i}=-1.466+4.698x_1+1.883x_2+x_1x_2+0.01x_1^2-4.21x_2^2 \tag{10}$$

The standard deviations of Eqs. 7, 8, 9, and 10 have been calculated to be ±25.7%, ±34.2%, ±34.2%, and ±29.5%, respectively.

All the reduced properties, for example, $P_{R_i}$, $x_1$ or $x_2$, attain a value of 1.0 for a homogeneous bed. Clearly, such an essential condition is satisfied by Eqs. 8 and 9, which show the values of $P_{R_i}$ to be 1.0 at $x_1=x_2=1.0$. Nevertheless, they have been discarded for having excessive standard deviations. Equation 10 is also ignored for having higher standard deviation than that of Eq. 7 and failing to satisfy the essential condition, that is, at $x_1=x_2=1.0$, $P_{R_i}=1.0$. The acceptable correlation is, therefore, Eq. 7, which produces an error of 6.1 percent only in meeting the essential condition. The correlation plot for Eq. 7 has been shown in Figure 2, and a histogram showing the frequency distribution of percentage errors is depicted in Figure 3. For a situation where the solid particles differ only in size and not in density, the ratio $\overline{Ar}/Ar_i^*$ becomes equal to the cube of the ratio of the average to the individual particle diameter, that is, $(\bar{d}_p/d_{p_i}^*)^3=(d_R)_i^3$ and hence Eq. 7 is modified into:

$$P_{R_i}=0.939(d_R)_i^{1.746}(\rho_{B_R})_i^{-1.365} \tag{11}$$

The ranges of the groups in Eq. 7 are as follows:

$$0.018\leq(Ar_R)_i\leq10.114,$$

and

$$0.9225\leq(\rho_{B_R})_i\leq1.1552$$

The effect of mixing coarse to fine particles or fine to coarse particles on a particular reduced system property $Re_{t_R}$ is shown in Figures 4 and 5, respectively. The variation of $\overline{Ar}/Ar_f^*$ or $\overline{Ar}/Ar_c^*$ is shown with $\overline{Re}_t/Re_{tf}^*$ or $\overline{Re}_t/Re_{tc}^*$ at constant $\rho_{B_R}$. It is clear that $\overline{Ar}$ is higher than $Ar_f^*$ (hence, $\overline{Ar}/Ar_f^*$ is greater than 1.0) in the former case, while $\overline{Ar}$ is less than $Ar_c^*$ (hence, $\overline{Ar}/Ar_c^*$ is less than 1.0) in the latter case. Hence, all the points in Figure 4 originate from $\overline{Ar}/Ar_f^*=1.0$, and those in Figure 5 terminate at $\overline{Ar}/Ar_c^*=1.0$. The variation of other reduced system properties, for example, $u_{osf_R}$, $u_{be_R}$, $u_{mf_R}$ and $u_{msf_R}$ with the reduced Archimedes number at constant reduced bulk densities has been presented in Figure 6. The scanning of the points indicates the possibility of the existence of a mean line, having a slope of 0.582, passing through the point (1, 1), which is the essential condition for the present postulate.

The major advantage of the present model over the average model proposed recently (Ganguly, 1989) lies in the possibility of evaluating any dynamic property ($u_t$, $u_{osf}$, $u_{mf}$, and so on) of a polydispersed system from a knowledge of the same dynamic property of a monodispersed system and the physical

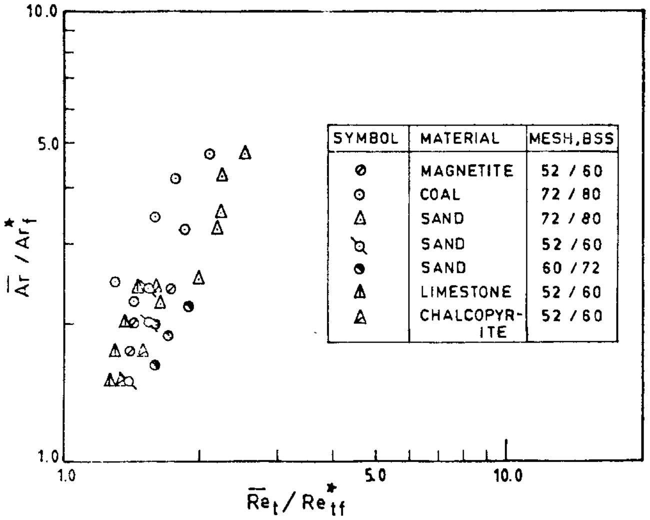


**Figure 4. Variation of $\overline{Ar}/Ar_f^*$ with $\overline{Re}_t/Re_{tf}^*$ at constant $\bar{\rho}_B/\rho_{B_f}^*$ (effect of addition of coarse to fine particles on $\overline{Re}_t/Re_{tf}^*$).**

characteristics of the solids and fluid. The second advantage of the proposed model is that, a single generalized correlation, that is, Eq. 7, is capable of calculating the average system (or dynamic) property provided the same property of any of the species is known beforehand.

## Conclusions

The concept of the classical theory of corresponding states of thermodynamics has been utilized in the field of fluidization to draw a generalized correlation for estimating system properties such as, $u_{mf}$, $u_{me}$, $u_{osf}$, $u_{msf}$, or $u_t$. According to the proposed model, any system property can be determined for a polydispersed system by means of Eq. 7 if the same property for a monodispersed system is known. Conversely, the system property of any of the solid species constituting the heterogeneous system can be determined if the same property of the polydispersed system is known. The degree of departure from the unicomponent systems, both with respect to system as well as static properties, is expressed in terms of the corresponding reduced properties, which are related according to Eq. 7 to provide standard deviation of ±25.7% based on the experimental data of $u_{mf}$, $u_{me}$, $u_{be}$, $u_{osf}$, $u_{msf}$ or $u_t$.

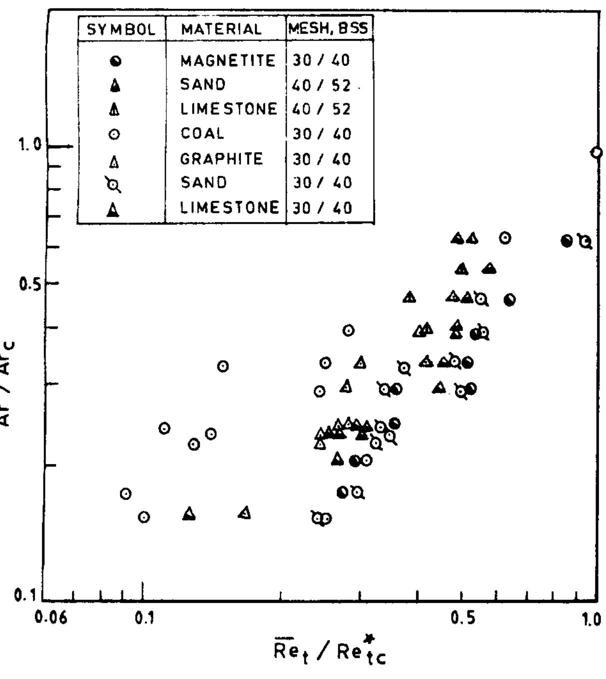


**Figure 5. Variation of $\overline{Ar}/Ar_c^*$ with $\overline{Re}_t/Re_{tc}^*$ at constant $\bar{\rho}_B/\rho_{B_c}^*$ (effect of mixing fines with coarse particles on $\overline{Re}_t/Re_{tc}^*$).**

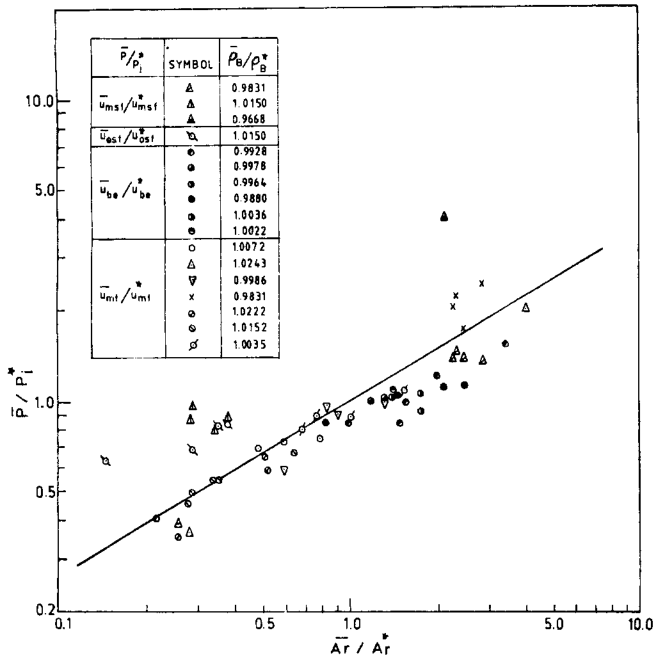


**Figure 6. Variation of reduced system properties: $u_{osf_R}$, $u_{be_R}$, $u_{mf_R}$ and $u_{msf_R}$ with reduced Archimedes number at constant reduced bulk density.**

## Notation

$a_1, a_2$ = exponents in Eq. 2
$A$ = coefficient in Eq. 2
$\overline{Ar}$ = Archimedes number for polydispersed bed $= \bar{d}_p^3 g\rho_f(\bar{\rho}_s - \rho_f)/\mu_f^2$, dimensionless
$Ar_i^*$ = Archimedes number for monodispersed system of species $i$ $[= d_{p_i}^{*3} g\rho_f(\rho_{s_i}^* - \rho_f)/\mu_f^2]$, dimensionless
$Ar_R$ = reduced Archimedes number
$Ar_c^*$ = Archimedes number for monodispersed system of coarse particles, dimensionless
$Ar_f^*$ = Archimedes number for monodispersed system of fine particles, dimensionless
$(Ar_R)_i$ = reduced Archimedes number with respect to species $i = \overline{Ar}/Ar_i^*$
$b_1$ = exponent in Eq. 3
$B$ = coefficient in Eq. 3
$c_1, c_2$ = exponents in Eq. 4
$C$ = coefficient in Eq. 4
$\bar{d}_p$ = average particle diameter for polydispersed system = $1/\Sigma(x_i/d_{p_i}^*)$, m

$d^*_{p_i}$ = particle diameter for monodispersed system of species $i$, m
$(d_R)_i$ = reduced particle diameter with respect to species $i$ = $\bar{d}_p/d^*_{p_i}$, dimensionless
$D$, $D_1$ to $D_5$ = coefficients in Eq. 5
$D_t$ = diameter of the fluidizing column, m
$g$ = acceleration due to gravity, m/s$^2$
$h_p$ = height of the packed bed below the constraint in semifluidization, m
$h_s$ = static bed height, m
$L_e$ = expanded bed height, m
$L_s$ = height of the solid bed (at $\epsilon = 0$), m
$P$ = existing pressure, N/m$^2$ or system property
$\bar{P}$ = average system property for polydispersed system
$\Delta P_a$ = actual pressure drop across the bed, N/m$^2$
$P_c$ = critical pressure, N/m$^2$
$P^*_i$ = system property for monodispersed system of species $i$
$P_R$ = reduced pressure ($= P/P_c$), dimensionless
$P_R$ = reduced system property, dimensionless
$(P_R)_i$ = reduced system property with respect to species $i$ = $\bar{P}/P^*_i$, dimensionless
$Re_{me}$ = Reynolds number at minimum elutriation condition, dimensionless
$Re_t$ = Reynolds number at free settling condition, dimensionless
$\overline{Re}_t$ = free settling Reynolds number for polydispersed system, dimensionless
$Re^*_{tc}$ = free settling Reynolds number for monodispersed system comprising coarse particles, dimensionless
$Re^*_{tf}$ = free settling Reynolds number for monodispersed system comprising fine particles, dimensionless
$T$ = existing temperature, K
$T_c$ = critical temperature, K
$T_R$ = reduced temperature, dimensionless
$u_{be}$ = beginning elutriation velocity, m/s
$u_{be_R}$ = reduced beginning elutriation velocity, dimensionless
$u_f$ = superficial fluid velocity, m/s
$u_{me}$ = minimum elutriation velocity, m/s
$u_{mf}$ = minimum fluidization velocity, m/s
$u_{mf_R}$ = reduced minimum fluidization velocity, dimensionless
$u_{msf}$ = maximum semifluidization velocity, m/s
$u_{msf_R}$ = reduced maximum semifluidization velocity, dimensionless
$u_{osf}$ = onset of semifluidization velocity, m/s
$u_{osf_R}$ = reduced onset of semifluidization velocity, dimensionless
$u_t$ = terminal settling velocity, m/s
$u_{t_R}$ = reduced settling velocity, dimensionless
$W_f$ = weight of solids charged into the fluidizing column, kg
$x_i$ = mass fraction of the individual species, $i$, dimensionless
$x_1$ = $(Ar_R)_i$, dimensionless
$x_2$ = $(\rho_{B_R})_i$, dimensionless

### Greek letters

$\epsilon$ = bed voidage, dimensionless
$\mu_f$ = fluid viscosity, Pa·s
$\bar{\rho}_B$ = bulk density of polydispersed system = $\Sigma(x_i\rho^*_{B_i})$, kg/m$^3$
$\rho_{B_R}$ = reduced bulk density, dimensionless
$(\rho_{B_R})_i$ = reduced bulk density with respect to species $i$, = $\bar{\rho}_B/\rho^*_{B_i}$, dimensionless
$\rho^*_{B_i}$ = bulk density of monodispersed system comprising species $i$, kg/m$^3$
$\rho_f$ = fluid density, kg/m$^3$
$\bar{\rho}_s$ = average solid density of the polydispersed system = $\Sigma(x_i\rho^*_{s_i})$, kg/m$^3$
$\rho^*_{s_i}$ = solid density of monodispersed system comprising species, $i$, kg/m$^3$

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